

Estimating Dynamic Dependence in Sequential Contests: Evidence from Professional Tennis

Nicolas Legrand¹ Jeremy Rosen²

¹Virginia Tech

²Georgetown University

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Motivation

- In sequential contests from sports to politics, it's an open question if events are independent and identically distributed (IID)
 - Momentum can influence outcomes of such contests (see Knight & Schiff 2010 on Kerry's early 2004 primary victories over Dean)
 - If momentum exists in sports like tennis, it may also exist in more consequential but harder to measure real-world contests (e.g. politics)
 - See Palacios-Huerta (2025) on sports as a natural laboratory
- Klaassen & Magnus (2001) found points in tennis are not IID, though it's still a good approximation. But two areas remain for exploration:
 - 1992–1995 Wimbledon might not be 100% representative of tennis due to fast grass courts (Martin 2010) and time period (Tignor 2017)
 - While highly promising, their iterated FGLS estimator for dynamic panels (Hsiao 1986) hasn't been applied to data before or after 2001

Research Questions

- 1 In a more representative dataset, are points in tennis IID?
 - Our dataset spans from 2014–2017 and contains matches from ATP and WTA pro tournaments from Grand Slams to Challengers
 - Also contains all three main surfaces (clay, grass, and hard)
- 2 How good is KM's FGLS estimator for dynamic panels?
 - In tennis data, cross sections are server-receiver-match pairs a , and time periods are points t . $y_{at} = 1$ if server wins point, and $y_{a,t-1}$ is an independent variable. If its parameter is > 0 , there's a hot hand
 - OLS and fixed effects are inconsistent (Nickell 1981)
 - In 2001, econometricians used GMM estimators (Arellano & Bond 1991, Blundell & Bond 1998) that are consistent but inefficient
 - Bias-corrected FE are more efficient but didn't exist in 2001
 - Therefore, KM implemented iterated FGLS estimator inspired by Hsiao (1986). Should applied econometricians use this estimator?

- **KM (2001):** $y_{at} = x'_a \beta + z'_{at} \delta + u_{at}$, where $u_{at} = \eta_a + \epsilon_{at}$
 - **Subscripts:** Match consists of players $(\mathcal{A}, \mathcal{B})$ subscripted as (a, b) . Therefore, a = server-receiver-match pair, and t = point in a
 - **Variables:** $y = \mathbb{1}(\text{Server Wins Point})$, x = observed player quality, z = dynamics (e.g. \tilde{y}_{-1}), η = unobserved quality, and ϵ = random error
- **Quality:** $x'_a = (1, R_a - R_b, R_a + R_b)$, where $R_a = 8 - \log_2(\text{RANK}_a)$
 - $R_a - R_b$ and $R_a + R_b$ are quality difference and sum of players
 - x is time-invariant, so not all estimators can estimate $\beta' = (\beta_0, \beta_1, \beta_2)$
- **Dynamics:** $z'_{at} = (\tilde{y}_{a,t-1} x'_a, d_{at} x'_a, \text{imp}_{at} x'_a)$
 - $\tilde{y}_{-1} = y_{-1} \mathbb{1}(d = 1)$, where $d = \mathbb{1}(\text{First Point in Service Game})$
 - $\text{imp} = P(\text{Server Wins Match} | \text{Wins Point}) - P(\text{Wins Match} | \text{Loses Point})$
 - Conditional probabilities depend on player quality and score (KM 2003)

Hypotheses and Data

- **Dynamic Parameters:** $\delta_i = \delta_{i0} + \delta_{i1}(R_a - R_b) + \delta_{i2}(R_a + R_b)$
for $i = 1, 2, 3 \Rightarrow \delta' = (\delta_{10}, \delta_{11}, \delta_{12}, \delta_{20}, \delta_{21}, \delta_{22}, \delta_{30}, \delta_{31}, \delta_{32})$
 - **IID:** $\delta = 0$, and **Homogeneity:** $\delta_{i1} = \delta_{i2} = 0$ for $i = 1, 2, 3$
 - In practice, estimate only significant $(\delta_{10}, \delta_{12}, \delta_{30}, \delta_{31}, \delta_{32})$
- **Data:** Who wins each point, player quality, and importance of point (calculated by computer program) for each server-receiver-match pair
 - **KM (2001):** 1992–1995 Wimbledon show courts
 - **LR (2025):** 2014–2017 Match Charting Project (Sackmann 2013) containing clay, grass, and hard court matches of all different levels
 - Weight matches by inverse sampling percentages

▶ Weighting

▶ Summary Statistics

▶ OLS and FE

- **Model:** $y_{at} = x'_{at}\beta + z'_{at}\delta + u_{at}$, where $u_{at} = \eta_a + \epsilon_{at}$

- ① **Effects Correlated Across Players:**

$$\text{var}(\eta_a) = \text{var}(\eta_b) = \tau^2, \text{cov}(\eta_a, \eta_b) = \gamma$$

- ② **Observed and Unobserved Quality Uncorrelated:**

$$\text{cov}(x_a, \eta_a) = \text{cov}(x_a, \eta_b) = 0$$

- ③ **Dynamics Predetermined:**

$$\text{cov}(\epsilon_{at}, z_{as}) = 0 \quad (s \leq t), \quad \text{cov}(\epsilon_{at}, z_{bs}) = 0 \quad (s \leq S_{bt})$$

- ④ **Heteroskedasticity:** $\text{var}(\epsilon_{at}) \equiv \sigma_a^2$

- Variance matrix of errors $(u'_a, u'_b)'$ of match is:

$$\Omega_{ab} = \text{var} \begin{pmatrix} u_a \\ u_b \end{pmatrix} = \begin{pmatrix} \sigma_a^2 I_{T_a} + \tau^2 l_a l'_a & \gamma l_a l'_b \\ \gamma l_b l'_a & \sigma_b^2 I_{T_b} + \tau^2 l_b l'_b \end{pmatrix}.$$

- 1 Estimate δ with Anderson & Hsiao (1981) First-Difference IV
 - 1 First difference $y_{at} - y_{a,t-1} = (z'_{at} - z'_{a,t-1})\delta + (\epsilon_{at} - \epsilon_{a,t-1})$ no longer contains effect η_a . z_{at} correlated with $\epsilon_{a,t-1}$, so estimate δ with IV
 - 2 Instrument is $z_{a,t-1}$ (which contains $\tilde{y}_{a,t-2}$) because it's correlated with $z_{at} - z_{a,t-1}$ but not $\epsilon_{at} - \epsilon_{a,t-1}$. However, $z_{a,t-1}$ is often weak
- 2 Estimate β with Anderson & Hsiao (1982)
 - 1 Use $\hat{\delta}$ to set up $y_{at}^* = x_a'\beta + u_{at}^*$, where $y_{at}^* = y_{at} - z'_{at}\hat{\delta}$
 - 2 Estimate β by regressing y_{at}^* on x_a . $(\hat{\delta}', \hat{\beta}')$ consistent but inefficient
- 3 Use $(\hat{\delta}', \hat{\beta}')$ to estimate $(\tau^2, \gamma, \sigma_a^2)$, and obtain $\hat{\Omega}$ (Hsiao 1986)

► FDIV (AH 1981)

- To obtain more efficient $(\hat{\delta}', \hat{\beta}')'$, perform FGLS on model using $\hat{\Omega}$. Like random effects, but $\hat{\Omega}$ is consistent since $(\hat{\delta}', \hat{\beta}')'$ are consistent. First show GLS is consistent with known Ω . Need two assumptions:
 - 1 Observed and unobserved quality uncorrelated
 - 2 Absence of initial conditions, i.e. dynamic z_{at} doesn't depend on endogenous variables from before sample period like y_{a0} . Holds in tennis data because there are no points before the first point
- Under these assumptions, GLS is equivalent to normality-based pseudo-ML, which is consistent \Rightarrow GLS is consistent (KM 2001)
- After obtaining more efficient $(\hat{\delta}', \hat{\beta}')'$, find $(\hat{\tau}^2, \hat{\gamma}, \hat{\sigma}_a^2)$ to get new $\hat{\Omega}$. Repeat until convergence. Final $(\hat{\delta}', \hat{\beta}')'$ will be much more efficient than original Anderson & Hsiao (1981, 1982) estimates or GMM

FGLS: Hot Hand Smaller from 2014–2017

Table 1: FGLS (Klaassen & Magnus 2001) Estimation Results

	1992–1995 Wimbledon				2014–2017			
	Men's singles		Women's singles		Men's singles		Women's singles	
Constant (β_0)	.6456	(.0040)	.5596	(.0050)	.6448	(.0035)	.5606	(.0032)
Quality difference (β_1)	.0106	(.0014)	.0198	(.0017)	.0141	(.0012)	.0123	(.0012)
Quality sum (β_2)	.0035	(.0014)	.0040	(.0017)	-.0026	(.0012)	.0024	(.0013)
<i>Previous point $\tilde{y}_{a,t-1}$</i>								
× constant (δ_{10})	.0085	(.0041)	.0123	(.0058)	.0031	(.0034)	.0088	(.0031)
× quality sum (δ_{12})	-.0028	(.0014)	-.0052	(.0019)	.0016	(.0011)	-.0008	(.0013)
<i>Importance imp_{at}</i>								
× constant (δ_{30})	-.1304	(.0666)	-.1752	(.0779)	-.0866	(.0432)	-.0995	(.0445)
× quality difference (δ_{31})	.0394	(.0262)	.1405	(.0493)	-.0286	(.0182)	-.0067	(.0186)
× quality sum (δ_{32})	.0533	(.0231)	.0137	(.0241)	.0425	(.0152)	-.0044	(.0188)
<i>Random effects</i>								
Variance (τ^2)	.0031		.0026		.0045		.0044	
Correlation (γ/τ^2)	-.5353		-.8317		-.2865		-.6432	
<i>Wald tests</i>								
IID	22.9901	[.0003]	25.2179	[.0001]	18.6268	[.0023]	15.6825	[.0078]
Homogeneity	14.6005	[.0022]	16.1926	[.0010]	10.7493	[.0132]	.5708	[.9031]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Previous Point: FGLS Between OLS and FE

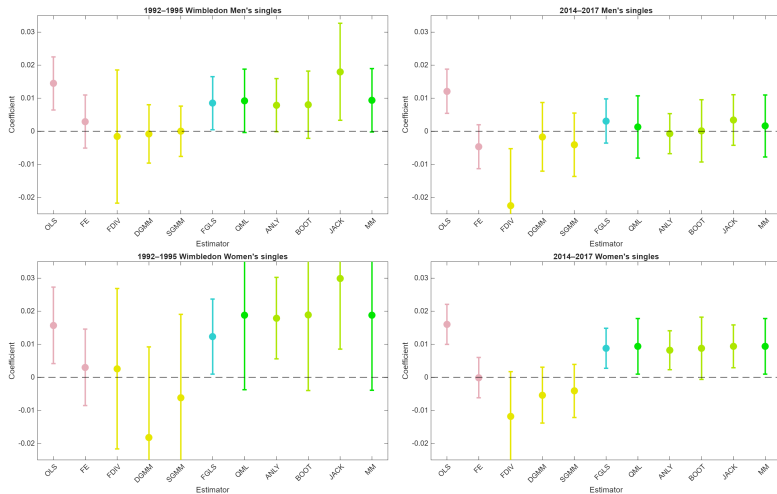


Figure 1: Comparison of Previous Point (δ_{10}) Estimates

Importance: FGLS Between OLS and FE

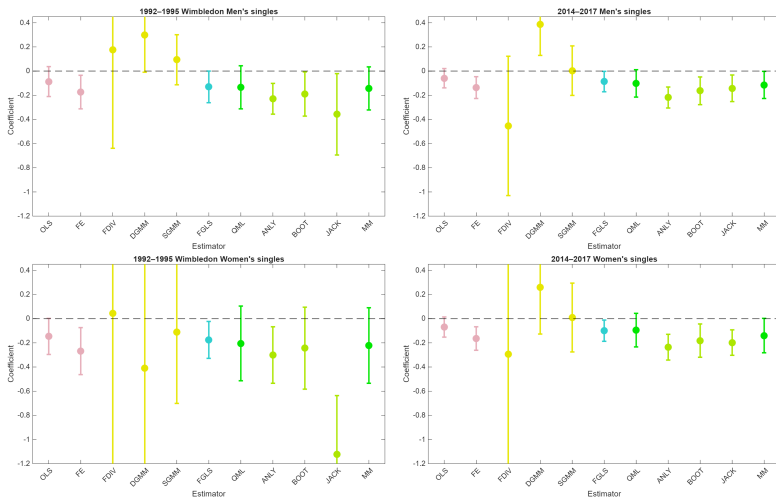


Figure 2: Comparison of Importance (δ_{30}) Estimates

IID: FGLS More Efficient Than GMM

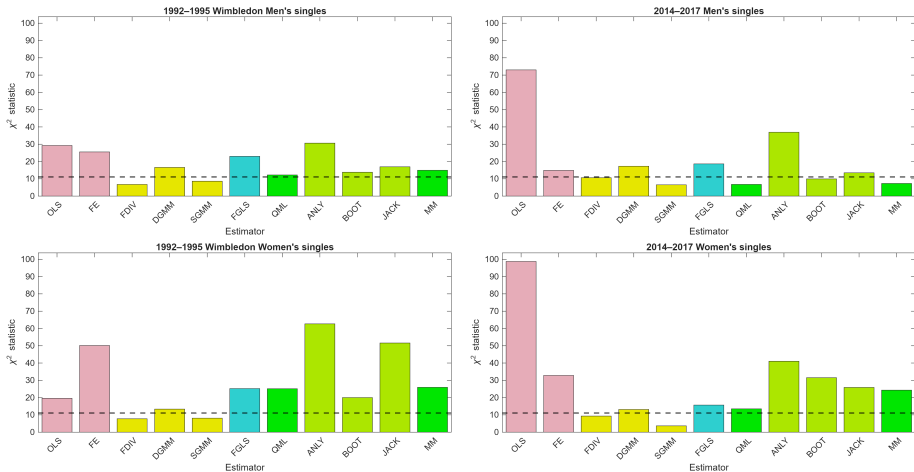


Figure 3: Comparison of IID Test (5 df) Results

Homogeneity: FGLS More Efficient Than GMM

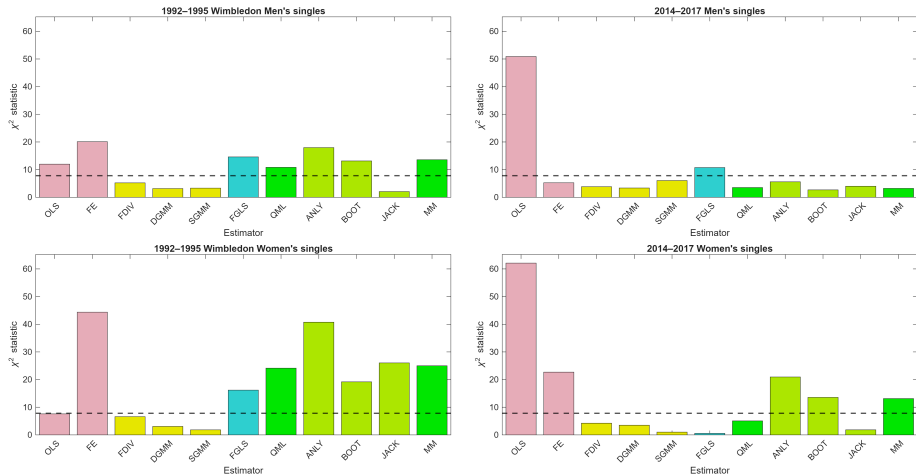


Figure 4: Comparison of Homogeneity Test (3 df) Results

Conclusion

★ Recall, our research questions are:

- 1 In a more representative dataset, are points in tennis IID?
- 2 How good is KM (2001)'s FGLS estimator for dynamic panels?

1 Like KM, we find points in tennis are not IID. However:

- Hot hand effect and receiver's edge on important points are smaller
- We don't reject no hot hand for men, or homogeneity for women
- Our parameters are not significantly different from theirs

2 FGLS is very good for dynamic panels

- Estimates always between OLS and uncorrected FE estimates
- Provided it's consistent, can quickly obtain efficient estimates (like BCFE) for predetermined and time-invariant variables (like GMM)
- "This approach may find application outside tennis as well" (KM). It's been over 20 years, but given its success here, we still hope so!

- **IID & Momentum Tennis Papers Pre-KM (2001):** Croucher (1981); Jackson & Mosurski (1997); Magnus & Klaassen (1999a,b,c)
- **IID & Momentum Tennis Papers Post-KM (2001):** Malueg & Yates (2010); Page & Coates (2017); Gauriot & Page (2019)
- **Hot Hand in Basketball:** Gilovich, Vallone & Tversky (1985); Miller & Sanjurjo (2018, 2021); Lantis & Nesson (2021, 2024)
 - LN (2024) find there's a hot hand iff players stay put between 3pt shots
 - **Upshot:** Hot hand likelier on fast courts as in KM (2001), since short points back then (Sackmann 2016) meant less time between serves

Appendix: Econometrics Literature

- **OLS & FE:** Inconsistent for dynamic panels!
- **GMM:** Consistent but inefficient!
 - **First-Difference IV:** Anderson & Hsiao (1981, 1982)
 - **Difference GMM:** Arellano & Bond (1991)
 - **System GMM:** Blundell & Bond (1998)
- **FGLS:** Hsiao (1986); Klaassen & Magnus (2001)
- **ML:** Hsiao, Pesaran & Tahmiscioglu (2002)
 - **Analytical:** Bruno (2005)
 - **Bootstrap:** Everaert & Pozzi (2007)
 - **Jackknife:** Dhaene & Jochmans (2015)
- **MM:** Breitung, Kripfganz & Hayakawa (2022)

Summary of Results

- ① In a more representative dataset, are points in tennis IID?
 - Like KM (2001), we reject IID at 1% level
 - Our hot hand parameter is smaller for both men (.0031 vs. .0085) and women (.0088 vs. .0123), and not significant for men
 - Receiver's edge on important points is less than KM's
 - Our parameters are not significantly different from theirs
- ② How good is KM's FGLS estimator for dynamic panels?
 - Very good! Estimates comparable to bias-corrected FE estimators, and always between OLS and uncorrected FE estimates
 - Fast estimates for predetermined and time-invariant variables
 - Provided FGLS is consistent, it's a very useful estimator

Appendix: Weighting

- **KM (2001):** Show courts biased in favor of better players (MK 1999a)
 - Tabulate matches by round, and seeded vs. non-seeded
 - Population is all 1992–1995 Wimbledon matches
- **LR (2025):** Sample of charted matches similarly biased
 - **Step 1:** Map round into equivalent Grand Slam round by ranking points
 - **Step 2:** Replace seeded vs. non-seeded with top 32 vs. non-top 32
 - **Step 3:** “Population” is all 2014–2017 Slam matches. If category doesn’t exist in population, combine it with nearest category by round

▶ Hypotheses and Data

▶ Match Equivalents

- **OLS:** Estimate $y_{at} = x'_a\beta + z'_{at}\delta + \epsilon_{at}$, ignoring effects η_a
 - $\hat{\delta}_{10}$ on $\tilde{y}_{a,t-1}$ biased up because WLOG if $\tilde{y}_{a,t-1} = 1$, then match a is more likely between strong servers. If so, y_{at} is more likely also = 1
 - $\hat{\delta}_{30}$ on imp_{at} biased up because important points are more common in matches between strong servers, since lopsided sets are less likely
- **FE:** Estimate time-demeaned $y_{at} - \bar{y}_a = (z'_{at} - \bar{z}'_a)\delta + (\epsilon_{at} - \bar{\epsilon}_a)$
 - z_{at} contains $\tilde{y}_{a,t-1}$, which is correlated with $\epsilon_{a,t-1}$, which is in $\bar{\epsilon}_a$. Therefore, $\mathbb{E}[(z'_{at} - \bar{z}'_a)(\epsilon_{at} - \bar{\epsilon}_a)] \neq 0 \Rightarrow$ dynamic panel bias
 - If $T \rightarrow \infty$, then bias $\rightarrow 0$. But for fixed T , bias $\not\rightarrow 0$, even if $N \rightarrow \infty$
 - If $\delta_{10} > 0$, then $\hat{\delta}_{10}$ biased down (Nickell 1981)

Appendix: FGLS: Few Discernible Patterns Across Surfaces

Table 2: FGLS Estimation Results for 2014–2017 Data by Surface

	Men's singles			Women's singles		
	Clay	Grass	Hard	Clay	Grass	Hard
Constant (β_0)	.6268 (.0076)	.6770 (.0111)	.6471 (.0042)	.5564 (.0069)	.5830 (.0081)	.5562 (.0040)
Quality difference (β_1)	.0143 (.0024)	.0131 (.0025)	.0146 (.0015)	.0143 (.0024)	.0117 (.0030)	.0118 (.0016)
Quality sum (β_2)	-.0065 (.0026)	-.0017 (.0034)	-.0021 (.0014)	.0021 (.0028)	.0092 (.0040)	.0014 (.0017)
<i>Previous point $\bar{y}_{a,t-1}$</i>						
× constant (δ_{10})	.0076 (.0069)	-.0020 (.0101)	.0021 (.0042)	.0140 (.0066)	.0272 (.0086)	.0062 (.0039)
× quality sum (δ_{12})	.0025 (.0024)	.0001 (.0031)	.0013 (.0014)	-.0033 (.0026)	-.0118 (.0042)	.0014 (.0016)
<i>Importance imp_{at}</i>						
× constant (δ_{30})	-.0368 (.0903)	.1088 (.1447)	-.1196 (.0522)	-.0782 (.0988)	-.2056 (.1237)	-.0714 (.0542)
× quality difference (δ_{31})	.0111 (.0405)	-.0519 (.0553)	-.0401 (.0216)	-.1267 (.0404)	.0439 (.0559)	.0182 (.0228)
× quality sum (δ_{32})	.0199 (.0320)	.0860 (.0418)	.0590 (.0190)	.0102 (.0387)	.0367 (.0597)	-.0138 (.0230)
<i>Random effects</i>						
Variance (τ^2)	.0044	.0036	.0043	.0044	.0032	.0043
Correlation (γ/τ^2)	-.0013	.0802	-.5233	-.7019	-1.0588	-.6458
<i>Wald tests</i>						
IID	3.0533 [.6918]	5.3527 [.3744]	20.6849 [.0009]	17.9767 [.0030]	23.5247 [.0003]	6.7501 [.2399]
Homogeneity	1.3265 [.7228]	5.3115 [.1504]	12.7166 [.0053]	11.4103 [.0097]	9.9378 [.0191]	1.9073 [.5919]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix: First-Difference IV, GMM, and Bias-Corrected FE

- **First-Difference IV (Anderson & Hsiao 1981):** See [▶ Slide 12](#)
- **Difference GMM (Arellano & Bond 1991):** Can we use more lags?
 - Using $z_{a,t-1}$ means we must drop observations where $t = 1$. If we add $z_{a,t-2}$, we must drop observations where $t = 2$, etc. Not feasible!
 - Instead use different instrument set for each t and estimate with GMM
- **System GMM (Blundell & Bond 1998):** Can we also estimate β ?
 - Get more efficient estimates by estimating system of two equations:
 - 1 First-difference equation with lagged levels as instruments
 - 2 Level equation with lagged differences as instruments
 - **Mean Stationarity:** Assume lagged differences uncorrelated with effects
- **Bias-Corrected FE:** Efficient estimates, but not always consistent for predetermined z_{at} or time-invariant x_a . See Appendix C for details

[▶ Previous Point Results](#)

[▶ Importance Results](#)

[▶ IID Results](#)

[▶ Homogeneity Results](#)

[▶ FDIV \(AH 1981\) Results](#)

[▶ DGMM \(AB 1991\) Results](#)

[▶ SGMM \(BB 1998\) Results](#)

Appendix A: Men's Match Equivalents

Table 3: 2014–2017 Men's Singles Match Equivalents by Ranking Points

Tournament Category	W	F	SF	QF	R16	R32	R64	R128	RR
Grand Slam	2000	1200	720	360	180	90	45	10	
ATP Masters 1000 (128)	1000	600	360	180	90	45	25	10	
ATP Masters 1000 (64)	1000	600	360	180	90	45	10		
ATP 500 Series	500	300	180	90	45	20			
ATP 250 Series	250	150	90	45	20	5			
Challenger \$125,000 +H	125	75	45	25	10	0			
Challenger \$125,000	110	65	40	20	9	0			
Challenger \$100,000	100	60	35	18	8	0			
Challenger \$75,000	90	55	33	17	8	0			
Challenger \$50,000	80	48	29	15	7	0			
Challenger \$35,000 +H	80	48	29	15	6	0			
ATP Finals	1100–1500	600–1000	200–600						0–400
ATP Challenger Tour Finals	95–125	45–75	15–45						0–30
Olympics	750	450	270–340	135	70	35	5		
Davis Cup	10–625	10–350	10–210	10–80	10				
Hopman Cup									

NOTE: The Olympics last awarded ranking points in 2012.

Appendix A: Women's Match Equivalents

Table 4: 2014–2017 Women's Singles Match Equivalents by Ranking Points

Tournament Category	W	F	SF	QF	R16	R32	R64	R128	RR
Grand Slam	2000	1300	780	430	240	130	70	10	
WTA Premier Mandatory (96)	1000	650	390	215	120	65	35	10	
WTA Premier Mandatory (64/60)	1000	650	390	215	120	65	10		
WTA Premier 5	900	585	350	190	105	60	1		
WTA Premier (56)	470	305	185	100	55	30	1		
WTA Premier (32)	470	305	185	100	55	1			
WTA International	280	180	110	60	30	1			
WTA 125K Series	160	95	57	29	15	1			
ITF \$100,000	140	85	50	25	13	1			
ITF \$60,000	80	48	29	15	8	1			
WTA Finals	1250–1500	830–1080	500–750						375–625
WTA Elite Trophy	620–700	360–440	160–240						80–160
WTA Tournament of Champions	380–450	185–255	110–180						75–145
Olympics	685	470	260–340	175	95	55	1		
Fed Cup									

NOTE: The Olympics last awarded ranking points in 2012.

Appendix A: Match Weights

Table 5: Number of Matches in 2014–2017 Sample and Population

Round	Sd-Sd			Sd-NSd			NSd-NSd		
	Sample	Population	Weight	Sample	Population	Weight	Sample	Population	Weight
Men's singles									
1	6	—		62	510	8.2258	66	514	7.7879
2	20	—	1.6462	66	428	6.4848	37	84	2.2703
3	39	107		32	122	3.8125	7	27	3.8571
4	74	81	1.0946	27	46	1.7037	13	1	
5	55	57	1.0364	11	7		2	0	.0667
6	42	32	.7619	1	0	.5833	0	0	—
7	13	16	1.2308	0	0	—	0	0	—
Women's singles									
1	23	—		109	511	4.6881	75	513	6.8400
2	29	—	.7241	108	388	3.5926	26	124	4.7692
3	64	84		70	124	1.7714	35	48	1.3714
4	50	61	1.22	35	60	1.7143	8	7	.875
5	76	42	.5526	12	18	1.5	1	4	4
6	23	20	.8696	8	12	1.5	0	0	—
7	9	13	1.4444	2	3	1.5	0	0	—

NOTE: In the sample, seeded = ranked in the top 32.

Appendix A: Summary Statistics

Table 6: Summary Statistics

	1992–1995 Wimbledon				2014–2017			
	Men's singles		Women's singles		Men's singles		Women's singles	
<i>Current point</i> $\tilde{y}_{a,t}$.6073	(.5251)	.5057	(.5507)	.5798	(.5526)	.5196	(.5383)
Quality difference	-.0112	(2.7946)	-.0123	(2.6453)	-.0034	(2.5788)	.0022	(2.5117)
Quality sum	.7065	(2.8039)	1.0356	(2.8135)	.6842	(2.8878)	.5537	(2.3505)
<i>Previous point</i> $\tilde{y}_{a,t-1}$								
× constant	.4826	(.5288)	.4165	(.5357)	.4619	(.5455)	.4279	(.5275)
× quality sum	.3802	(2.0436)	.4935	(1.9864)	.3494	(2.0815)	.2613	(1.6166)
<i>First point in game</i> d_{at}								
× constant (δ_{20})	.1533	(.3740)	.1398	(.3694)	.1480	(.3769)	.1431	(.3664)
<i>Importance</i> imp_{at}								
× constant	.0255	(.0350)	.0285	(.0431)	.0354	(.0456)	.0431	(.0409)
× quality difference	.0005	(.0970)	-.0001	(.0771)	-.0005	(.1170)	-.0003	(.1297)
× quality sum	.0111	(.1217)	.0276	(.1671)	.0093	(.1630)	.0191	(.1403)
N_{points} $N_{matches}$	57,319	258	28,979	223	82,797	573	102,938	763

NOTE: Standard deviations are in parentheses.

Appendix A: Summary Statistics by Surface

Table 7: Summary Statistics for 2014–2017 Data by Surface

	Clay		Men's singles Grass		Hard		Clay		Women's singles Grass		Hard	
<i>Current point</i> $\bar{y}_{a,t}$.5665	(.5564)	.5861	(.5760)	.5824	(.5491)	.5174	(.5386)	.5325	(.5504)	.5136	(.5405)
Quality difference	-.0066	(2.2900)	.0207	(3.4108)	-.0016	(2.5553)	.0061	(2.6620)	.0032	(2.6484)	.0011	(2.4176)
Quality sum	.6381	(2.8177)	.7372	(3.2468)	.6578	(2.8462)	.5123	(2.5013)	.5244	(1.9790)	.5589	(2.3422)
<i>Previous point</i> $\bar{y}_{a,t-1}$												
× constant	.4531	(.5458)	.4603	(.5650)	.4645	(.5434)	.4264	(.5269)	.4324	(.5390)	.4241	(.5286)
× quality sum	.2999	(2.0111)	.3987	(2.3699)	.3389	(2.0490)	.2376	(1.7052)	.2640	(1.3980)	.2615	(1.6100)
<i>Importance</i> imp_{at}												
× constant	.0380	(.0459)	.0300	(.0426)	.0352	(.0458)	.0430	(.0399)	.0364	(.0396)	.0443	(.0416)
× quality difference	-.0009	(.1101)	.0021	(.1143)	-.0006	(.1207)	.0007	(.1283)	.0006	(.1092)	-.0008	(.1339)
× quality sum	.0073	(.1616)	.0022	(.1804)	.0104	(.1593)	.0189	(.1531)	.0128	(.1118)	.0206	(.1414)
N_{points} $N_{matches}$	20,229	141	9,184	53	53,384	379	22,820	172	13,211	100	66,907	491

NOTE: Standard deviations are in parentheses.

▶ Hypotheses and Data

Appendix B: OLS: Inconsistent and Biased Up

Table 8: OLS Estimation Results

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
Constant (β_0)	.6400 (.0036)	.5570 (.0048)	.6387 (.0031)	.5555 (.0030)
Quality difference (β_1)	.0106 (.0008)	.0202 (.0012)	.0143 (.0007)	.0147 (.0008)
Quality sum (β_2)	.0037 (.0013)	.0039 (.0016)	-.0028 (.0010)	.0027 (.0013)
<i>Previous point</i> $\tilde{y}_{a,t-1}$				
× constant (δ_{10})	.0145 (.0041)	.0157 (.0059)	.0121 (.0034)	.0160 (.0031)
× quality sum (δ_{12})	-.0029 (.0014)	-.0051 (.0019)	.0014 (.0011)	-.0012 (.0013)
<i>Importance</i> imp_{at}				
× constant (δ_{30})	-.0875 (.0634)	-.1470 (.0757)	-.0594 (.0407)	-.0694 (.0425)
× quality difference (δ_{31})	-.0294 (.0238)	-.0170 (.0422)	-.1017 (.0164)	-.1272 (.0162)
× quality sum (δ_{32})	.0468 (.0222)	.0126 (.0234)	.0494 (.0142)	-.0064 (.0180)
<i>Wald tests</i>				
IID	29.2796 [.0000]	19.6088 [.0015]	72.9962 [.0000]	98.7666 [.0000]
Homogeneity	11.9846 [.0074]	7.6061 [.0549]	50.8934 [.0000]	62.1154 [.0000]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix B: FE: Inconsistent for Fixed T and Biased Down

Table 9: FE Estimation Results

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
<i>Previous point $\tilde{y}_{a,t-1}$</i>				
× constant (δ_{10})	.0029 (.0041)	.0030 (.0059)	-.0047 (.0034)	-.0001 (.0031)
× quality sum (δ_{12})	-.0025 (.0014)	-.0052 (.0020)	.0015 (.0011)	-.0006 (.0013)
<i>Importance imp_{at}</i>				
× constant (δ_{30})	-.1743 (.0711)	-.2690 (.0989)	-.1356 (.0461)	-.1635 (.0495)
× quality difference (δ_{31})	.0803 (.0276)	.3548 (.0584)	.0293 (.0196)	.0965 (.0204)
× quality sum (δ_{32})	.0563 (.0249)	.0018 (.0301)	.0235 (.0164)	.0003 (.0208)
<i>Wald tests</i>				
IID	25.5306 [.0001]	50.1330 [.0000]	14.8639 [.0110]	32.8313 [.0000]
Homogeneity	20.1028 [.0002]	44.3690 [.0000]	5.2625 [.1536]	22.6947 [.0000]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix B: FGLS: Unweighted Comparable to Weighted

Table 10: Unweighted FGLS Estimation Results

	1992–1995 Wimbledon				2014–2017			
	Men's singles		Women's singles		Men's singles		Women's singles	
Constant (β_0)	.6462	(.0040)	.5642	(.0050)	.6413	(.0035)	.5657	(.0032)
Quality difference (β_1)	.0105	(.0010)	.0199	(.0014)	.0144	(.0012)	.0123	(.0012)
Quality sum (β_2)	.0036	(.0014)	.0066	(.0019)	-.0008	(.0010)	.0013	(.0012)
<i>Previous point $\tilde{y}_{a,t-1}$</i>								
× constant (δ_{10})	.0098	(.0040)	.0045	(.0058)	.0053	(.0034)	.0042	(.0031)
× quality sum (δ_{12})	-.0025	(.0014)	-.0065	(.0021)	.0000	(.0010)	-.0012	(.0011)
<i>Importance imp_{at}</i>								
× constant (δ_{30})	-.0098	(.0635)	-.0842	(.0823)	-.0552	(.0436)	-.0813	(.0436)
× quality difference (δ_{31})	.0322	(.0223)	.0746	(.0398)	-.0023	(.0200)	-.0244	(.0182)
× quality sum (δ_{32})	.0402	(.0223)	-.0272	(.0261)	.0218	(.0126)	.0026	(.0167)
<i>Random effects</i>								
Variance (τ^2)	.0027		.0025		.0043		.0043	
Correlation (γ/τ^2)	-.3277		-.5668		-.3077		-.5620	
<i>Wald tests</i>								
IID	16.5335	[.0055]	15.2714	[.0093]	8.9878	[.1096]	9.2049	[.1012]
Homogeneity	10.6724	[.0136]	13.9339	[.0030]	3.1506	[.3690]	2.9082	[.4060]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix B: FDIV: Consistent but Biased Down

Table 11: First-Difference IV (Anderson-Hsiao 1981)

	1992–1995 Wimbledon				2014–2017			
	Men's singles		Women's singles		Men's singles		Women's singles	
<i>Previous point $\tilde{y}_{a,t-1}$</i>								
× constant (δ_{10})	-.0016	(.0103)	.0026	(.0124)	-.0225	(.0088)	-.0118	(.0069)
× quality sum (δ_{12})	.0001	(.0019)	-.0048	(.0028)	.0026	(.0016)	-.0014	(.0019)
<i>First point in game d_{at}</i>								
× constant (δ_{20})	.0031	(.0141)	.0163	(.0164)	-.0282	(.0118)	-.0032	(.0089)
<i>Importance imp_{at}</i>								
× constant (δ_{30})	.1757	(.4153)	.0433	(1.0549)	-.4526	(.2940)	-.2941	(.4807)
× quality difference (δ_{31})	-.0368	(.0965)	.1986	(.3080)	.0539	(.0724)	.1977	(.1099)
× quality sum (δ_{32})	.1480	(.0711)	.1634	(.1702)	.0587	(.0475)	.0043	(.0917)
<i>Wald tests</i>								
IID	6.7427	[.2405]	7.7510	[.1705]	10.6374	[.0591]	9.3233	[.0968]
Homogeneity	5.2177	[.1565]	6.5993	[.0858]	3.8284	[.2806]	4.2412	[.2366]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix B: Difference GMM: Consistent but Biased Down

Table 12: Difference GMM (Arellano & Bond 1991)

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
<i>Previous point $\tilde{y}_{a,t-1}$</i>				
× constant (δ_{10})	-0.0008 (.0045)	-0.0182 (.0140)	-0.0017 (.0053)	-0.0054 (.0043)
× quality sum (δ_{12})	-0.0011 (.0015)	-0.0017 (.0038)	.0020 (.0012)	-0.0010 (.0020)
<i>First point in game $d_{a,t}$</i>				
× constant (δ_{20})	.0066 (.0036)	-0.0057 (.0069)	.0057 (.0038)	.0011 (.0025)
<i>Importance imp_{at}</i>				
× constant (δ_{30})	.2975 (.1567)	-.4100 (.5283)	.3861 (.1314)	.2582 (.1971)
× quality difference (δ_{31})	.0279 (.0783)	.3313 (.3253)	-.0522 (.0670)	.1518 (.0844)
× quality sum (δ_{32})	.0345 (.0420)	.1453 (.1196)	.0290 (.0362)	-.0434 (.0698)
<i>Wald tests</i>				
IID	16.5905 [.0053]	13.3373 [.0204]	17.2782 [.0040]	13.1394 [.0221]
Homogeneity	3.1623 [.3673]	3.0736 [.3804]	3.3621 [.3391]	3.5085 [.3197]

NOTE: Standard errors are in parentheses; p values are in square brackets.

► FDIV (AH 1981), DGMM (AB 1991), SGMM (BB 1998)

Appendix B: System GMM: Consistent but Biased Down

Table 13: System GMM (Blundell & Bond 1998)

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
Constant (β_0)	.6422 (.0056)	.5657 (.0153)	.6444 (.0066)	.5611 (.0088)
Quality difference (β_1)	.0098 (.0016)	.0186 (.0030)	.0133 (.0020)	.0113 (.0020)
Quality sum (β_2)	.0031 (.0023)	.0033 (.0032)	-.0029 (.0018)	.0036 (.0032)
<i>Previous point $\tilde{y}_{a,t-1}$</i>				
× constant (δ_{10})	.0000 (.0039)	-.0062 (.0129)	-.0041 (.0049)	-.0041 (.0041)
× quality sum (δ_{12})	-.0011 (.0015)	-.0037 (.0037)	.0023 (.0012)	-.0014 (.0020)
<i>First point in game $d_{a,t}$</i>				
× constant (δ_{20})	.0050 (.0030)	-.0027 (.0066)	.0012 (.0034)	-.0009 (.0024)
<i>Importance imp_{at}</i>				
× constant (δ_{30})	.0939 (.1056)	-.1107 (.3017)	.0031 (.1046)	.0083 (.1453)
× quality difference (δ_{31})	.0198 (.0619)	.1290 (.1507)	-.0517 (.0499)	-.0273 (.0458)
× quality sum (δ_{32})	.0394 (.0346)	.0127 (.0642)	.0433 (.0320)	-.0199 (.0496)
<i>Wald tests</i>				
IID	8.6095 [.1257]	8.1013 [.1507]	6.5023 [.2604]	3.6408 [.6022]
Homogeneity	3.3174 [.3452]	1.8311 [.6082]	6.0494 [.1092]	1.0079 [.7993]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix C: Previous Point: FGLS Between OLS and FE

Table 14: Comparison of Previous Point (δ_{10}) Estimates

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
OLS	.0145 (.0041)	.0157 (.0059)	.0121 (.0034)	.0160 (.0031)
FE	.0029 (.0041)	.0030 (.0059)	-.0047 (.0034)	-.0001 (.0031)
FDIV (AH 1981)	-.0016 (.0103)	.0026 (.0124)	-.0225 (.0088)	-.0118 (.0069)
DGMM (AB 1991)	-.0008 (.0045)	-.0182 (.0140)	-.0017 (.0053)	-.0054 (.0043)
SGMM (BB 1998)	.0000 (.0039)	-.0062 (.0129)	-.0041 (.0049)	-.0041 (.0041)
FGLS (KM 2001)	.0085 (.0041)	.0123 (.0058)	.0031 (.0034)	.0088 (.0031)
QML (HPT 2002)	.0092 (.0049)	.0188 (.0115)	.0013 (.0048)	.0094 (.0043)
ANLY (B 2005a)	.0079 (.0041)	.0179 (.0063)	-.0007 (.0031)	.0082 (.0030)
BOOT (EP 2007)	.0080 (.0052)	.0189 (.0117)	.0001 (.0048)	.0088 (.0048)
JACK (DJ 2015)	.0180 (.0075)	.0299 (.0109)	.0034 (.0039)	.0094 (.0033)
MM (BKH 2022)	.0094 (.0049)	.0188 (.0116)	.0016 (.0048)	.0094 (.0043)

NOTE: Standard errors are in parentheses.

▶ Previous Point Figure

▶ QML (HPT 2002)

▶ ANLY (B 2005a)

▶ BOOT (EP 2007)

▶ JACK (DJ 2015)

▶ MM (BKH 2022)

Appendix C: Importance: FGLS Between OLS and FE

Table 15: Comparison of Importance (δ_{30}) Estimates

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
OLS	-.0875 (.0634)	-.1470 (.0757)	-.0594 (.0407)	-.0694 (.0425)
FE	-.1743 (.0711)	-.2690 (.0989)	-.1356 (.0461)	-.1635 (.0495)
FDIV (AH 1981)	.1757 (.4153)	.0433 (1.0549)	-.4526 (.2940)	-.2941 (.4807)
DGMM (AB 1991)	.2975 (.1567)	-.4100 (.5283)	.3861 (.1314)	.2582 (.1971)
SGMM (BB 1998)	.0939 (.1056)	-.1107 (.3017)	.0031 (.1046)	.0083 (.1453)
FGLS (KM 2001)	-.1304 (.0666)	-.1752 (.0779)	-.0866 (.0432)	-.0995 (.0445)
QML (HPT 2002)	-.1333 (.0909)	-.2055 (.1575)	-.1019 (.0583)	-.0944 (.0710)
ANLY (B 2005a)	-.2290 (.0651)	-.3010 (.1194)	-.2184 (.0440)	-.2362 (.0543)
BOOT (EP 2007)	-.1897 (.0935)	-.2437 (.1731)	-.1629 (.0586)	-.1820 (.0703)
JACK (DJ 2015)	-.3572 (.1720)	-1.1221 (.2478)	-.1427 (.0565)	-.1981 (.0539)
MM (BKH 2022)	-.1430 (.0907)	-.2228 (.1592)	-.1153 (.0574)	-.1411 (.0726)

NOTE: Standard errors are in parentheses.

► Importance Figure

► QML (HPT 2002)

► ANLY (B 2005a)

► BOOT (EP 2007)

► JACK (DJ 2015)

► MM (BKH 2022)

Appendix C: IID: FGLS More Efficient Than GMM

Table 16: Comparison of IID Test Results

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
OLS	29.2796 [.0000]	19.6088 [.0015]	72.9962 [.0000]	98.7666 [.0000]
FE	25.5306 [.0001]	50.1330 [.0000]	14.8639 [.0110]	32.8313 [.0000]
FDIV (AH 1981)	6.7427 [.2405]	7.7510 [.1705]	10.6374 [.0591]	9.3233 [.0968]
DGMM (AB 1991)	16.5905 [.0053]	13.3373 [.0204]	17.2782 [.0040]	13.1394 [.0221]
SGMM (BB 1998)	8.6095 [.1257]	8.1013 [.1507]	6.5023 [.2604]	3.6408 [.6022]
FGLS (KM 2001)	22.9901 [.0003]	25.2179 [.0001]	18.6268 [.0023]	15.6825 [.0078]
QML (HPT 2002)	12.2230 [.0319]	25.1289 [.0001]	6.7464 [.2402]	13.4969 [.0191]
ANLY (B 2005a)	30.6305 [.0000]	62.6533 [.0000]	36.9393 [.0000]	41.0433 [.0000]
BOOT (EP 2007)	13.8019 [.0169]	20.0125 [.0012]	9.9265 [.0773]	31.5070 [.0000]
JACK (DJ 2015)	16.9361 [.0020]	51.5826 [.0000]	13.4683 [.0194]	25.8734 [.0001]
MM (BKH 2022)	14.9262 [.0107]	25.9405 [.0001]	7.2950 [.1996]	24.3105 [.0002]

NOTE: *P* values are in square brackets.

▶ IID Figure

▶ QML (HPT 2002)

▶ ANLY (B 2005a)

▶ BOOT (EP 2007)

▶ JACK (DJ 2015)

▶ MM (BKH 2022)

Appendix C: Homogeneity: FGLS More Efficient Than GMM

Table 17: Comparison of Homogeneity Test Results

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
OLS	11.9846 [.0074]	7.6061 [.0549]	50.8934 [.0000]	62.1154 [.0000]
FE	20.1028 [.0002]	44.3690 [.0000]	5.2625 [.1536]	22.6947 [.0000]
FDIV (AH 1981)	5.2177 [.1565]	6.5993 [.0858]	3.8284 [.2806]	4.2412 [.2366]
DGMM (AB 1991)	3.1623 [.3673]	3.0736 [.3804]	3.3621 [.3391]	3.5085 [.3197]
SGMM (BB 1998)	3.3174 [.3452]	1.8311 [.6082]	6.0494 [.1092]	1.0079 [.7993]
FGLS (KM 2001)	14.6005 [.0022]	16.1926 [.0010]	10.7493 [.0132]	.5708 [.9031]
QML (HPT 2002)	10.8334 [.0127]	24.1300 [.0000]	3.5302 [.3169]	5.0695 [.1668]
ANLY (B 2005a)	17.9709 [.0004]	40.7474 [.0000]	5.6040 [.1325]	20.9429 [.0001]
BOOT (EP 2007)	13.1400 [.0043]	19.1837 [.0003]	2.6797 [.4437]	13.5814 [.0035]
JACK (DJ 2015)	2.0609 [.3568]	26.0286 [.0000]	4.0000 [.2615]	1.8365 [.6070]
MM (BKH 2022)	13.5825 [.0035]	24.9914 [.0000]	3.2263 [.3580]	13.1301 [.0044]

NOTE: *P* values are in square brackets.

▶ Homogeneity Figure

▶ QML (HPT 2002)

▶ ANLY (B 2005a)

▶ BOOT (EP 2007)

▶ JACK (DJ 2015)

▶ MM (BKH 2022)

Appendix D: Quasi-Maximum Likelihood FE (HPT 2002)

- **Intuition:** While First-Difference OLS is inconsistent, First-Difference ML is consistent if we can write likelihood for all t . Not trivial!
 - First difference $\Delta y_{at} = \Delta z'_{at} \delta + \Delta \epsilon_{at}$ (1) undefined for $t = 1$ because Δz_{a1} contains $\Delta \tilde{y}_{a,0}$, which contains undefined $\tilde{y}_{a,-1}$. Solution:
 - $\Delta y_{a1} = b + \sum_{s=1}^{\min(T_a)} \Delta z'_{as} \pi_s + \nu_{a1}$ (2), where $\text{var}(\nu_{a1}) = \omega \sigma_\epsilon^2$ (and $b = 0$)
 - For $t \geq 2$, likelihood contains (1). For $t = 1$, it contains (2)
 - Like FGLS, start with initial consistent $\hat{\delta}$. Use $\hat{\delta}$ to estimate variance matrix Ω , which contains $(\omega, \sigma_\epsilon^2)$. Then use $\hat{\Omega}$ to estimate better $\hat{\delta}$ with minimum distance. To maximize likelihood, iterate until convergence
- **Limitation:** All z_{at} except $\tilde{y}_{a,t-1}$ must be exogenous
 - FGLS doesn't require exogenous z_{at} , but it requires absence of initial conditions, which is stronger than restricting $b = 0$ in (2)
 - Unlike QML, FGLS can estimate β . Williams, Allison & Moral-Benito (2018) ML can also estimate β and handle predetermined z_{at} . But it's too computationally intensive for our data, where $T \gg 10$

▶ Previous Point Results

▶ Importance Results

▶ IID Results

▶ Homogeneity Results

▶ QML (HPT 2002) Results

Appendix D: Analytic Bias-Corrected FE (B 2005a)

- **Intuition:** Assuming $|\delta_{10}| < 1$, all z_{at} except $\tilde{y}_{a,t-1}$ are exogenous, and homoskedasticity (i.e. $\text{var}(\epsilon_{at}) = \sigma_\epsilon^2$), we can estimate analytic bias term B and subtract it from $\hat{\delta}^{fe}$ to get unbiased $\hat{\delta}^{anly}$
 - See Bruno (2005) for expression for B ; too long to reproduce here
 - To estimate B , need consistent $\hat{\delta}_{10}$ and $\hat{\sigma}_\epsilon^2$. Use First-Difference IV
 - Unlike previous versions, Bruno's allows for unbalanced panels
- **Limitation:** While $|\delta_{10}| < 1$, other two assumptions don't hold
 - $\hat{\delta}_{10}^{anly}$ between $\hat{\delta}_{10}^{ols}$ and $\hat{\delta}_{10}^{fe}$, but $\hat{\delta}_{30}^{anly} < \text{both } \hat{\delta}_{30}^{ols} \text{ and } \hat{\delta}_{30}^{fe}$
 - IID and homogeneity test statistics probably too high
 - Estimates still better than First-Difference IV and GMM

▶ Previous Point Results

▶ Importance Results

▶ IID Results

▶ Homogeneity Results

▶ ANLY (B 2005a) Results

Appendix D: Bootstrap Bias-Corrected FE (EP 2007)

- **Intuition:** $\hat{\delta}^{bc}$ is unbiased estimate of δ if data generated from it yield FE estimate that matches FE estimate yielded by actual data. Steps:
 - 1 Run FE on actual data to get $\hat{\delta}^{fe}$, and guess $\delta_{(0)}^* = \hat{\delta}^{fe}$
 - 2 Using $\delta_{(0)}^*$, calculate variance of residuals for each cross-section
 - 3 Conditioning on $y_{a,1}$ and all z_{at} except $\tilde{y}_{a,t-1}$, simulate J (e.g. 250) bootstrap samples using $\delta_{(0)}^*$ and calculated variances of residuals
 - 4 For each bootstrap sample, run FE to get $\hat{\delta}_j^b$, obtain correction $\omega_{(0)} = \hat{\delta}^{fe} - 1/J \sum_{j=1}^J \hat{\delta}_j^b$, and update $\delta_{(1)}^* = \delta_{(0)}^* + \omega_{(0)}$
 - 5 Iterate until $\omega_{(k)} \approx 0$. Then $\hat{\delta}^{bc} = \delta_{(k)}^*$, and bootstrap SEs
- **Limitation:** All z_{at} except $\tilde{y}_{a,t-1}$ must be exogenous
 - Since x_a is time invariant, we can correctly simulate $\tilde{y}_{a,t-1}x_a'$
 - imp_{at} is function of time-invariant player quality and the score, so we can (with difficulty) correctly simulate it and its interactions $imp_{at}x_a'$

Appendix D: Jackknife Bias-Corrected FE (DJ 2015)

- **Intuition:** Sending $T \rightarrow \infty$ would make FE bias disappear. Instead shrink T , making bias worse. Then send $\hat{\delta}^{spj}$ in opposite direction
 - Dhaene & Jochmans (2015) shrink sample with split-panel jackknife (e.g. first and second halves of each server-receiver-match pair)
 - **Simplest Case:** $\hat{\delta}^{spj} = \arg \max_{\delta} \left\{ 2\hat{\ell}(\delta) - \frac{1}{2} [\hat{\ell}_{S_1}(\delta) + \hat{\ell}_{S_2}(\delta)] \right\}$
 - Though model is linear, assume normal errors and estimate FE with ML
 - T Odd: Split sample both possible ways and take average of likelihoods
 - Unbalanced panel is union of balanced panels, so take weighted average of likelihoods. Ultimately an average of an average of an average
- **Limitation:** Given unbalanced panel and interactions, must drop server-receiver-match pairs a lacking enough others with same T_a
 - Estimates still comparable to FGLS and other BCFE estimators
 - Like FGLS, benefit of jackknife is z_{at} need not be exogenous

▶ Previous Point Results

▶ Importance Results

▶ IID Results

▶ Homogeneity Results

▶ JACK (DJ 2015) Results

Appendix D: MM Bias-Corrected FE (BKH 2022)

- **Intuition:** Previous BCFE estimators estimate bias term and subtract it from $\hat{\delta}^{fe}$. MM BCFE corrects bias “at the source” (Kripfganz & Breitung 2022), i.e. the moment conditions, as follows:

$$\hat{\delta}^{mm} = \arg \min_{\delta} \left(\sum_{a=1}^A m_a(\delta) \right)' \left(\sum_{a=1}^A m_a(\delta) \right), \text{ where:}$$

$$m_a(\delta) = \frac{1}{T_a} \sum_{t=1}^{T_a} \left[\begin{pmatrix} y_{a,t-1} - \bar{y}_{a,-1} \\ z_{at} - \bar{z}_a \end{pmatrix} - \begin{pmatrix} \frac{T_a}{T_a-1} b_a(\delta_{10})(u_{at} - \bar{u}_a) \\ 0 \end{pmatrix} \right] u_{at}, \text{ where:}$$

$$b_a(\delta_{10}) = -\frac{1}{T_a^2} \sum_{t=0}^{T_a-2} \sum_{s=0}^t \delta_{10}^s, \text{ such that: } \mathbb{E}[m_a(\delta)] = 0$$

- **Limitation:** All z_{at} except $\tilde{y}_{a,t-1}$ must be exogenous
 - Simulations show bias is small even with predetermined z (Breitung, Kripfganz & Hayakawa 2022), as evident in Table 22
 - Unlike Analytic BCFE, allows for heteroskedasticity (i.e. $\mathbb{E}[\epsilon_{at}] = \sigma_a^2$)

▶ Previous Point Results

▶ Importance Results

▶ IID Results

▶ Homogeneity Results

▶ MM (BKH 2022) Results

Appendix E: QML FE: Comparable to FGLS

Table 18: QML FE (Hsiao, Pesaran & Tahmiscioglu 2002)

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
<i>Previous point $\tilde{y}_{a,t-1}$</i>				
× constant (δ_{10})	.0092 (.0049)	.0188 (.0115)	.0013 (.0048)	.0094 (.0043)
× quality sum (δ_{12})	-.0024 (.0019)	-.0052 (.0034)	.0018 (.0013)	-.0001 (.0015)
<i>Importance imp_{at}</i>				
× constant (δ_{30})	-.1333 (.0909)	-.2055 (.1575)	-.1019 (.0583)	-.0944 (.0710)
× quality difference (δ_{31})	.0676 (.0328)	.3380 (.0702)	.0176 (.0304)	.0612 (.0273)
× quality sum (δ_{32})	.0498 (.0313)	-.0007 (.0426)	.0275 (.0181)	-.0012 (.0268)
<i>Wald tests</i>				
IID	12.2230 [.0319]	25.1289 [.0001]	6.7464 [.2402]	13.4969 [.0191]
Homogeneity	10.8334 [.0127]	24.1300 [.0000]	3.5302 [.3169]	5.0695 [.1668]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix E: Analytic BCFE: Comparable to FGLS

Table 19: Analytic Bias-Corrected FE (Bruno 2005)

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
<i>Previous point $\tilde{y}_{a,t-1}$</i>				
× constant (δ_{10})	.0079 (.0041)	.0179 (.0063)	-.0007 (.0031)	.0082 (.0030)
× quality sum (δ_{12})	-.0024 (.0014)	-.0055 (.0018)	.0016 (.0012)	-.0006 (.0014)
<i>Importance imp_{at}</i>				
× constant (δ_{30})	-.2290 (.0651)	-.3010 (.1194)	-.2184 (.0440)	-.2362 (.0543)
× quality difference (δ_{31})	.0799 (.0344)	.3211 (.0562)	.0297 (.0226)	.0889 (.0204)
× quality sum (δ_{32})	.0571 (.0267)	-.0005 (.0345)	.0239 (.0157)	.0007 (.0215)
<i>Wald tests</i>				
IID	30.6305 [.0000]	62.6533 [.0000]	36.9393 [.0000]	41.0433 [.0000]
Homogeneity	17.9709 [.0004]	40.7474 [.0000]	5.6040 [.1325]	20.9429 [.0001]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix E: Bootstrap BCFE: Comparable to FGLS

Table 20: Bootstrap Bias-Corrected FE (Everaert & Pozzi 2007)

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
<i>Previous point $\tilde{y}_{a,t-1}$</i>				
× constant (δ_{10})	.0080 (.0052)	.0189 (.0117)	.0001 (.0048)	.0088 (.0048)
× quality sum (δ_{12})	-.0025 (.0018)	-.0053 (.0034)	.0016 (.0014)	-.0006 (.0016)
<i>Importance imp_{at}</i>				
× constant (δ_{30})	-.1897 (.0935)	-.2437 (.1731)	-.1629 (.0586)	-.1820 (.0703)
× quality difference (δ_{31})	.0771 (.0336)	.3373 (.0809)	.0237 (.0292)	.0947 (.0262)
× quality sum (δ_{32})	.0581 (.0306)	.0006 (.0489)	.0224 (.0187)	-.0005 (.0281)
<i>Wald tests</i>				
IID	13.8019 [.0169]	20.0125 [.0012]	9.9265 [.0773]	31.5070 [.0000]
Homogeneity	13.1400 [.0043]	19.1837 [.0003]	2.6797 [.4437]	13.5814 [.0035]

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix E: Jackknife BCFE: Comparable to FGLS

Table 21: Split-Panel Jackknife Bias-Corrected FE (Dhaene & Jochmans 2015)

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
<i>Previous point $\tilde{y}_{a,t-1}$</i>				
× constant (δ_{10})	.0180 (.0075)	.0299 (.0109)	.0034 (.0039)	.0094 (.0033)
× quality sum (δ_{12})	-.0034 (.0024)	-.0133 (.0036)	.0021 (.0014)	-.0000 (.0014)
<i>Importance imp_{at}</i>				
× constant (δ_{30})	-.3572 (.1720)	-1.1221 (.2478)	-.1427 (.0565)	-.1981 (.0539)
× quality difference (δ_{31})	.0055 (.0653)	.3640 (.1258)	.0227 (.0242)	.0313 (.0232)
× quality sum (δ_{32})		.1634 (.1049)	.0246 (.0209)	-.0017 (.0227)
<i>Wald tests</i>				
IID	16.9361 [.0020]	51.5826 [.0000]	13.4683 [.0194]	25.8734 [.0001]
Homogeneity	2.0609 [.3568]	26.0286 [.0000]	4.0000 [.2615]	1.8365 [.6070]
N_{points} $N_{matches}$	18,251 94	8,674 77	65,614 490	96,598 727

NOTE: Standard errors are in parentheses; p values are in square brackets.

Appendix E: MM BCFE: Comparable to FGLS

Table 22: MM Bias-Corrected FE (Breitung, Kripfganz & Hayakawa 2022)

	1992–1995 Wimbledon		2014–2017	
	Men's singles	Women's singles	Men's singles	Women's singles
<i>Previous point $\tilde{y}_{a,t-1}$</i>				
× constant (δ_{10})	.0094 (.0049)	.0188 (.0116)	.0016 (.0048)	.0094 (.0043)
× quality sum (δ_{12})	-.0024 (.0019)	-.0054 (.0034)	.0015 (.0013)	-.0006 (.0015)
<i>Importance imp_{at}</i>				
× constant (δ_{30})	-.1430 (.0907)	-.2228 (.1592)	-.1153 (.0574)	-.1411 (.0726)
× quality difference (δ_{31})	.0788 (.0319)	.3508 (.0714)	.0295 (.0271)	.1013 (.0283)
× quality sum (δ_{32})	.0550 (.0313)	.0023 (.0432)	.0231 (.0183)	-.0022 (.0275)
<i>Wald tests</i>				
IID	14.9262 [.0107]	25.9405 [.0001]	7.2950 [.1996]	24.3105 [.0002]
Homogeneity	13.5825 [.0035]	24.9914 [.0000]	3.2263 [.3580]	13.1301 [.0044]

NOTE: Standard errors are in parentheses; p values are in square brackets.

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